*16. January 2026*

To gain full marks you need to justify all the answers, not only to calculate them.

Problem 1.
There is a square platform with a side length of 1 meter (dashed square in the middle) and there are stairs leading down from each side of the platform. Each stair is 1 meter wide. Michal and Vítek coloured the top side of each stair. Michal used the colour green and Vítek used the colour orange. You can see the top view of the final result in the picture. Who used more colour?
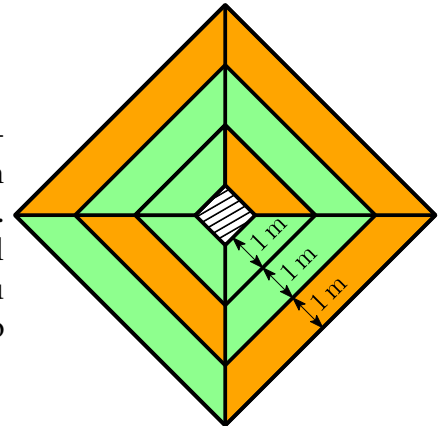
Problem 2.
Let ABC be an equilateral triangle. Points D, E, F, and G lie inside the sides of the triangle such that BD is perpendicular to AC, DE is perpendicular to BC, EF is perpendicular to BD, and FG is perpendicular to BC. Determine the ratio $|BG| : |GE| : |EC|$.

Problem 3.
Let $ABCD$ be a trapezium satisfying $AB \parallel CD$ and $AB \perp AD$. There is a point X on side AD satisfying $|AX| : |XD| = 2 : 1$ and $|\sphericalangle CXD| = |\sphericalangle AXB|$. Assume the area of triangle BCX is equal to 16 cm^2. Calculate the area of the trapezium $ABCD$.

Problem 4.
Let ABC be an acute scalene triangle. Denote by D and E the feet of the perpendiculars from A to BC and B to AC, respectively. Let X, Y be points such that $DXEY$ is a rhombus and X lies on segment AB. Assume that C lies inside $DXEY$ and that $|\sphericalangle CAY| = |\sphericalangle AYE|$. Find the measure of the angle $\sphericalangle ABE$.

Problem 5.
Let $ABCD$ be a square. Let E be a point on the line AD such that $|AE| = |BD|$ and D lies between the points A and E. The perpendicular bisector of the segment CE intersects the line CD at a point F. Show that the line EF is parallel to the diagonal BD. Does this hold even if A lies between D and E?

Problem 6.
Let $ABCD$ be a convex[1] quadrilateral with $|AB| = 2$, $|BC| = 1$ and $|\sphericalangle CDA| = 60°$. Find the maximum possible length of diagonal BD and justify why it cannot be larger.

Time: 4 *hours*

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[1] A polygon is called convex if all its interior angles are less than 180°.