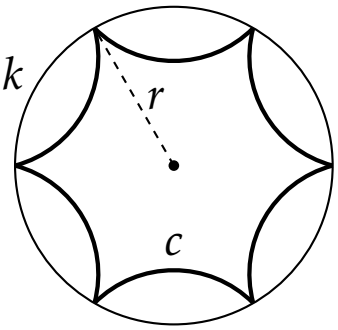
23. January 2026

To gain full marks you need to justify all the answers, not only to calculate them.

Problem 1.
Let k be a circle with radius r. Additionally there are 6 congruent quarter circles drawn as shown in the figure, forming a curve c. Determine the length of the curve c in terms of r.



Problem 2.
Let ABC be a triangle satisfying $|AB| = |AC|$. Let M be the midpoint of BC. Let N be a point such that $AM \parallel CN$ and $|AN| = |MN|$. Determine the ratio of the areas of the triangles ABC and ANC.

Problem 3.
Let ABC be an acute triangle. Let P and Q be the midpoints of minor arcs AC and AB of the circumcircle of ABC, respectively. Let R and S be points on the lines AP and AQ, respectively, such that $AC \perp CR$ and $AB \perp BS$. Prove that the incenter of the triangle ABC lies on the line RS.

Problem 4.
Let $ABCD$ be a convex quadrilateral such that there is a point P inside $ABCD$ for which $|AP| = |AB|$, $|DP| = |DC|$, $|\sphericalangle PBA| = 2|\sphericalangle PAD|$ and $|\sphericalangle PCD| = 2|\sphericalangle PDA|$. Let O be the circumcenter of triangle PBC and let M be the midpoint of OP. Prove that $|MA| = |MD|$.

Problem 5.
Let $ABCD$ be a convex quadrilateral such that there is a point E on the side AB satisfying $|\sphericalangle ADE| = |\sphericalangle DEA| = |\sphericalangle DCE|$ and $|\sphericalangle ECB| = |\sphericalangle BEC| = |\sphericalangle EDC|$. Prove that one of the common tangents to the incircles of triangles AED and BEC is parallel to the line CD.

Problem 6.
Let ABC be an acute scalene triangle with incenter I. Let M be the midpoint of arc BAC of the circumcircle of triangle ABC. Points K and L lie on segments BM and CM, respectively, such that AK is tangent to the circumcircle of triangle AIC and AL is tangent to the circumcircle of triangle AIB. Prove that the points K, L, I are collinear.

Time: 5 *hours*