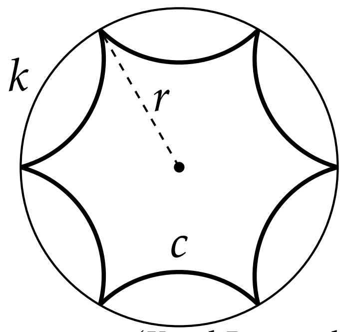
23. January 2026

**Problem 1.**
Let $k$ be a circle with radius $r$. Additionally there are 6 congruent quarter circles drawn as shown in the figure, forming a curve $c$. Determine the length of the curve $c$ in terms of $r$.



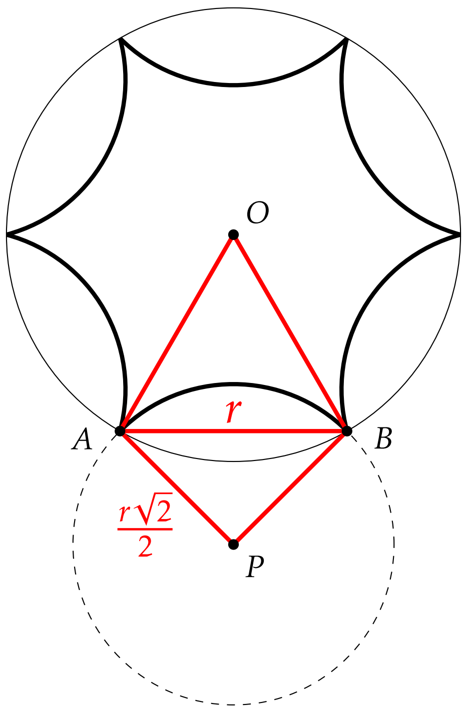

(Karel Pazourek)

**Solution**

Let $O$ be the center of the circle and let $A$ and $B$ be two adjacent common points of $c$ and $k$. Observe that $|OA| = |OB| = r$ and $|\sphericalangle AOB| = \frac{1}{6} \cdot 360^\circ = 60^\circ$, so the triangle $AOB$ is equilateral, hence $|AB| = r$.

Let $P$ be the center of the circle that contains the arc $AB$ of curve $c$. This arc is a quarter circle, so $|\sphericalangle APB| = \frac{1}{4} \cdot 360^\circ = 90^\circ$, hence $|PA| = |PB| = \frac{r\sqrt{2}}{2}$. We can now calculate the length of $c$:

$$\text{length}\,(c) = 6 \cdot \text{length}\,(\text{arc } AB) = 6 \cdot \frac{1}{4} \cdot 2\pi \frac{r\sqrt{2}}{2} = \frac{3\sqrt{2}}{2}\pi r.$$



**Problem 2.**
Let $ABC$ be a triangle satisfying $|AB| = |AC|$. Let $M$ be the midpoint of $BC$. Let $N$ be a point such that $AM \parallel CN$ and $|AN| = |MN|$. Determine the ratio of the areas of the triangles $ABC$ and $ANC$.

*(Marián Macko)*

**Solution**

Let $P$ be the midpoint of the segment $AM$. Triangle $AMN$ is isosceles with base $AM$. Therefore $N$ lies on the perpendicular bisector of segment $AM$, which is the line $PN$. Hence, we have that $AM \perp PN$.

Triangle $ABC$ is isosceles as well, therefore $AM \perp BC$ as $AM$ is the perpendicular bisector of $BC$. We also have that $CN \parallel AM$. Therefore $PMCN$ is a rectangle. We have

$$|NC| = |PM| = \frac{1}{2}|AM|.$$

We first calculate the area of triangle $ANC$:

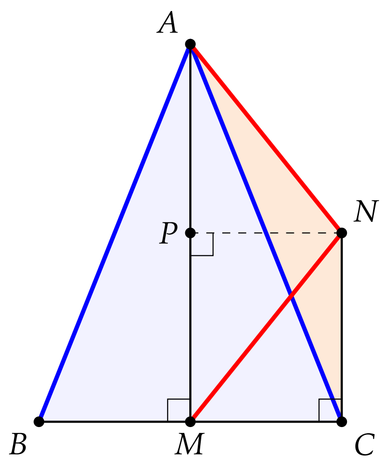
$$S_{ANC} = S_{AMCN} - S_{AMC}.$$

We aim to express each area in terms of $|AM|$ and $|BC|$.

We have:

$$S_{AMC} = \frac{1}{2}|AM| \cdot |CM| = \frac{1}{2}|AM| \cdot \frac{|BC|}{2} = \frac{1}{4}|AM| \cdot |BC|.$$

Quadrilateral $AMCN$ is a trapezium, therefore we can calculate its area using the lengths of the bases and the distance between them.




$$S_{AMCN} = \frac{1}{2}(|AM| + |NC|) \cdot |MC| = \frac{1}{2}(|AM| + \frac{1}{2}|AM|) \cdot \frac{|BC|}{2} = \frac{3}{8}|AM| \cdot |BC|.$$

And therefore

$$S_{ANC} = S_{AMCN} - S_{AMC} = \frac{3}{8}|AM| \cdot |BC| - \frac{1}{4}|AM| \cdot |BC| = \frac{1}{8}|AM| \cdot |BC|.$$

Finally, $S_{ABC} = \frac{1}{2}|AM| \cdot |BC|$ and therefore

$$\frac{S_{ABC}}{S_{ANC}} = \frac{\frac{1}{2}|AM| \cdot |BC|}{\frac{1}{8}|AM| \cdot |BC|} = 4.$$

**Problem 3.**
Let $ABC$ be an acute triangle. Let $P$ and $Q$ be the midpoints of minor arcs $AC$ and $AB$ of the circumcircle of $ABC$, respectively. Let $R$ and $S$ be points on the lines $AP$ and $AQ$, respectively, such that $AC \perp CR$ and $AB \perp BS$. Prove that the incenter of the triangle $ABC$ lies on the line $RS$.

*(Patrik Bak)*

**Solution**

Let $I$ denote the incenter of the triangle $ABC$. We will show that $I$ is the foot of the perpendicular from $A$ to $RS$, i.e. that the angles $\measuredangle AIS$ and $\measuredangle AIR$ are right. It suffices to show this.

We will show that the quadrilaterals $AIBS$ and $AICR$ are cyclic. Let $\alpha$, $\beta$, $\gamma$ denote the corresponding angles of the triangle $ABC$.

Since $Q$ is the midpoint of the arc $AB$, we have $|QA| = |QB|$. Therefore

$$|\measuredangle SAB| = |\measuredangle QAB| = \frac{180^\circ - |\measuredangle AQB|}{2} = \frac{180^\circ - (180^\circ - |\measuredangle BCA|)}{2} = \gamma/2.$$

We may then compute $|\measuredangle ASB| = 90^\circ - |\measuredangle SAB| = 90^\circ - \gamma/2$.

Consider the triangle $AIB$. The angles $\measuredangle BAI$ and $\measuredangle ABI$ have measures $\alpha/2$ and $\beta/2$, respectively, so we get

$$|\measuredangle AIB| = 180^\circ - \alpha/2 - \beta/2 = 180^\circ + \gamma/2 - (\alpha + \beta + \gamma)/2 = 180^\circ + \gamma/2 - 180^\circ/2 = 90^\circ + \gamma/2.$$

Now, clearly, the point $S$ lies outside the triangle $ABC$ and $I$ lies inside, hence the points $I$ and $S$ lie on opposite sides of the line $AB$. Since

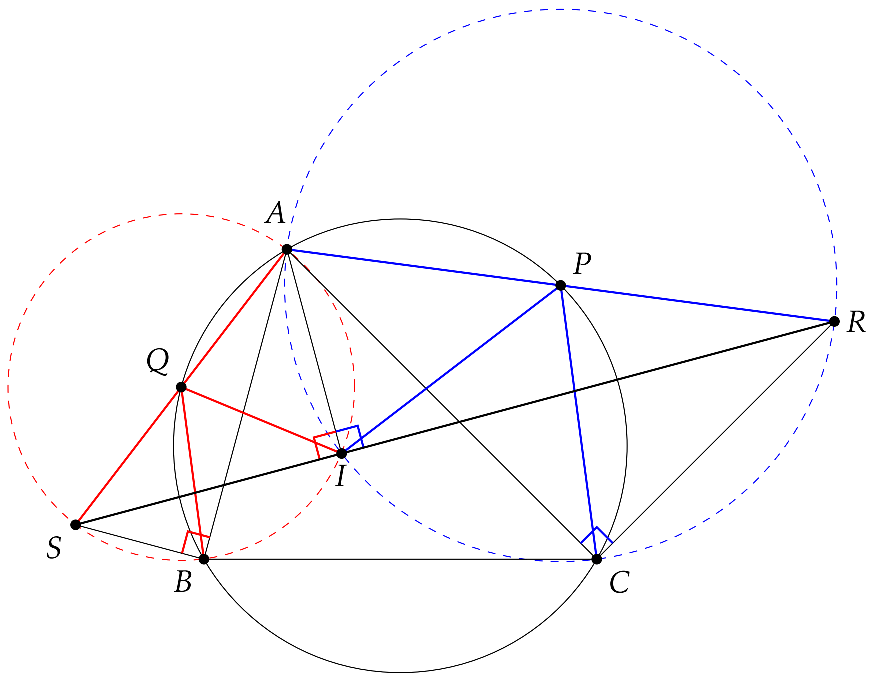
$$|\measuredangle ASB| + |\measuredangle AIB| = (90^\circ - \gamma/2) + (90^\circ + \gamma/2) = 180^\circ,$$

the points $A$, $I$, $B$ and $S$ lie on a circle. Similarly, the points $A$, $I$, $C$ and $R$ lie on a circle.

Since $|\measuredangle ABS| = 90^\circ$ and $|\measuredangle ACR| = 90^\circ$, the line segments $AS$ and $AR$ are diameters of the two circles, meaning

$$|\measuredangle AIS| + |\measuredangle AIR| = 90^\circ + 90^\circ = 180^\circ,$$

which implies that the three points $R$, $I$ and $S$ lie on a single line.




**Remark**. It is a well-known fact that the points $A$, $I$, $B$ lie on a circle with circumcenter $Q$. This is a useful property worth remembering. Using this, one can quickly see that $S$ also lies on this circle, as $AS$ is the diameter.

**Problem 4.**
Let $ABCD$ be a convex quadrilateral such that there is a point $P$ inside $ABCD$ for which $|AP| = |AB|$, $|DP| = |DC|$, $|\sphericalangle PBA| = 2|\sphericalangle PAD|$ and $|\sphericalangle PCD| = 2|\sphericalangle PDA|$. Let $O$ be the circumcenter of triangle $PBC$ and let $M$ be the midpoint of $OP$. Prove that $|MA| = |MD|$.

*(Michal Pecho)*

**Solution**

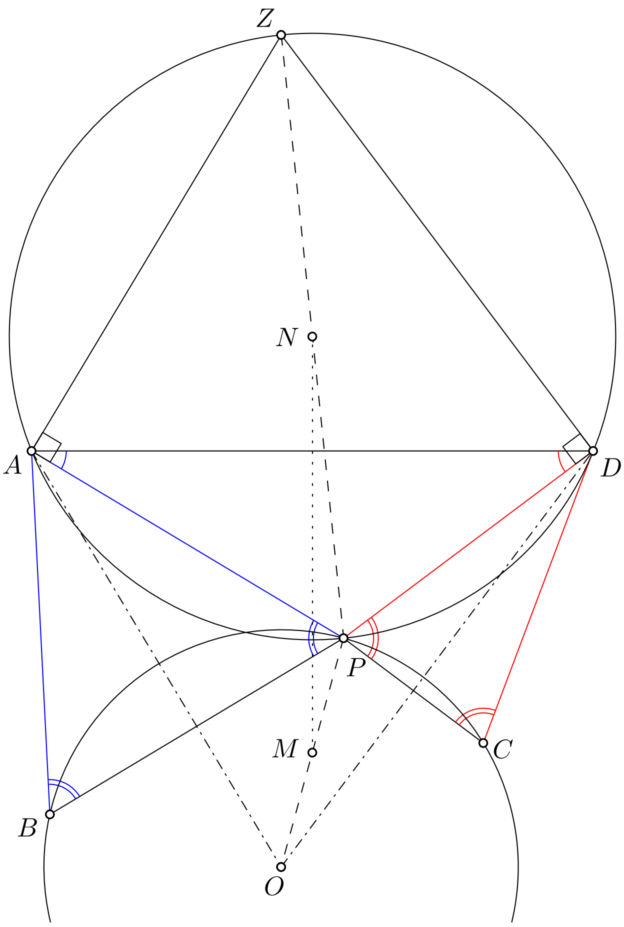
Points $A$ and $O$ lie on the perpendicular bisector of $BP$ and points $D$ and $O$ lie on the perpendicular bisector of $CP$. An angle chase yields:

$$\begin{aligned}|\sphericalangle OAD| &= |\sphericalangle OAP| + |\sphericalangle PAD| \\ &= 90^\circ - |\sphericalangle BPA| + |\sphericalangle PAD| \\ &= 90^\circ - |\sphericalangle PBA| + |\sphericalangle PAD| \\ &= 90^\circ - 2\cdot|\sphericalangle PAD| + |\sphericalangle PAD| \\ &= 90^\circ - |\sphericalangle PAD|.\end{aligned}$$

Analogously $|\sphericalangle ODA| = 90^\circ - |\sphericalangle PDA|$. Let $Z$ be the reflection of $O$ across $AD$. Note that $OZ \perp AD$. From symmetry, we get that $|\sphericalangle ZAD| = |\sphericalangle OAD|$ and $|\sphericalangle ZDA| = |\sphericalangle ODA|$. Therefore

$$|\sphericalangle ZAP| = |\sphericalangle ZAD| + |\sphericalangle PAD| = 90^\circ - |\sphericalangle PAD| + |\sphericalangle PAD| = 90^\circ,$$
$$|\sphericalangle ZDP| = |\sphericalangle ZDA| + |\sphericalangle PDA| = 90^\circ - |\sphericalangle PDA| + |\sphericalangle PDA| = 90^\circ,$$

which means that $Z$ lies on the circumcircle of triangle $PDA$ and $PZ$ is its diameter. Therefore, the midpoint $N$ of segment $PZ$ lies on the perpendicular bisector of $AD$. Since $MN \parallel OZ$ (as $MN$ is a midline in $\triangle OPZ$) and $OZ \perp AD$, we get that $MN \perp AD$. Thus, line $MN$ coincides with the perpendicular bisector of $AD$, so $M$ lies on it, which implies $|MA| = |MD|$.



**Solution II**

Let $P'$ and $P''$ be the reflections of $P$ across the line $AD$ and across the midpoint of $AD$, respectively. From the given angle conditions, we have

$$|\sphericalangle P'DP| = 2\cdot|\sphericalangle ADP| = |\sphericalangle DCP| = |\sphericalangle DPC|,$$

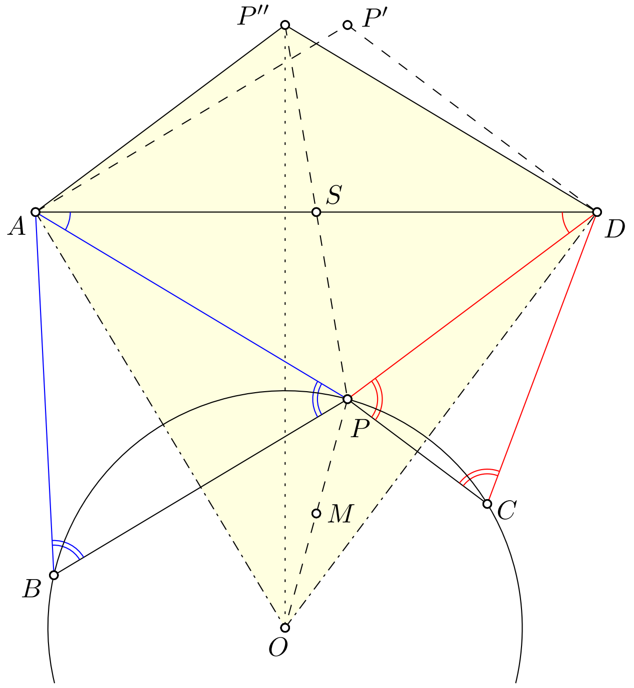
so $P'D \parallel PC$. Since the line $DO$ is the perpendicular bisector of $CP$, we have $DO \perp PC$. Therefore, $P'D \perp DO$. Analogously, we can see that $P'A \perp AO$.

Let $S$ be the midpoint of $AD$. Since $S$ is also the midpoint of $PP''$, the segment $SM$ is a midline in triangle $OPP''$, implying $SM \parallel P''O$. Thus, to show that $M$ lies on the perpendicular bisector of $AD$ (which passes through $S$), it suffices to show that $P''O \perp AD$.

We can now use the well-known fact that the diagonals of a quadrilateral $P''AOD$ are perpendicular if and only if the sums of the squares of its opposite sides are equal: $|P''D|^2 + |AO|^2 = |P''A|^2 + |DO|^2$. Using symmetry, we get:

$$|P''D|^2 + |AO|^2 = |P'A|^2 + |AO|^2 = |P'O|^2 = |P'D|^2 + |DO|^2 = |P''A|^2 + |DO|^2,$$

which implies $P''O \perp AD$ and completes the proof.



**Problem 5.**
Let $ABCD$ be a convex quadrilateral such that there is a point $E$ on the side $AB$ satisfying $|\sphericalangle ADE| = |\sphericalangle DEA| = |\sphericalangle DCE|$ and $|\sphericalangle ECB| = |\sphericalangle BEC| = |\sphericalangle EDC|$. Prove that one of the common tangents to the incircles of triangles $AED$ and $BEC$ is parallel to the line $CD$.

*(Patrik Bak)*

**Solution**

Let $I$ and $J$ be the incenters of triangles $AED$ and $BEC$, respectively. Notice that

$$|\sphericalangle EID| = 180° - |\sphericalangle IDE| - |\sphericalangle DEI| = 180° - |\sphericalangle DEA| = 180° - |\sphericalangle DCE|,$$

so the points $C, D, E, I$ are concyclic. Analogously, the points $C, D, E, J$ are concyclic. Hence, all five points $C, D, E, I, J$ are concyclic.

One of the common tangents of the two incircles is the line $AB$ – therefore another tangent line is the reflection of $AB$ with respect to the line $IJ$. Let $P$, $Q$ be the intersections of the reflected line with lines $ED$ and $EC$, respectively. We will prove that $PQ$ and $CD$ are parallel by showing that $|\sphericalangle EPQ| = |\sphericalangle EDC|$.

Let $R$, $S$ be the intersections of the line $IJ$ with lines $ED$ and $EC$, respectively. We calculate:

$$|\sphericalangle ERS| = |\sphericalangle RIE| + |\sphericalangle REI| = |\sphericalangle JIE| + |\sphericalangle DEI| = |\sphericalangle ECJ| + |\sphericalangle EDI| =$$

$$|\sphericalangle CEJ| + |\sphericalangle EJI| = |\sphericalangle SEJ| + |\sphericalangle SJE| = |\sphericalangle RSE|.$$

As $PQ$ is the reflection of $AB$ with respect to $IJ$, all three lines are parallel or concurrent.

a) We first look at the case where $PQ$, $AB$ and $IJ$ are all parallel. Then, by an angle chase, we get

$$|\sphericalangle EPQ| = |\sphericalangle ERS| = |\sphericalangle ESR| = |\sphericalangle BES| = |\sphericalangle BEC| = |\sphericalangle EDC|$$

so the lines $PQ$ and $CD$ are parallel.

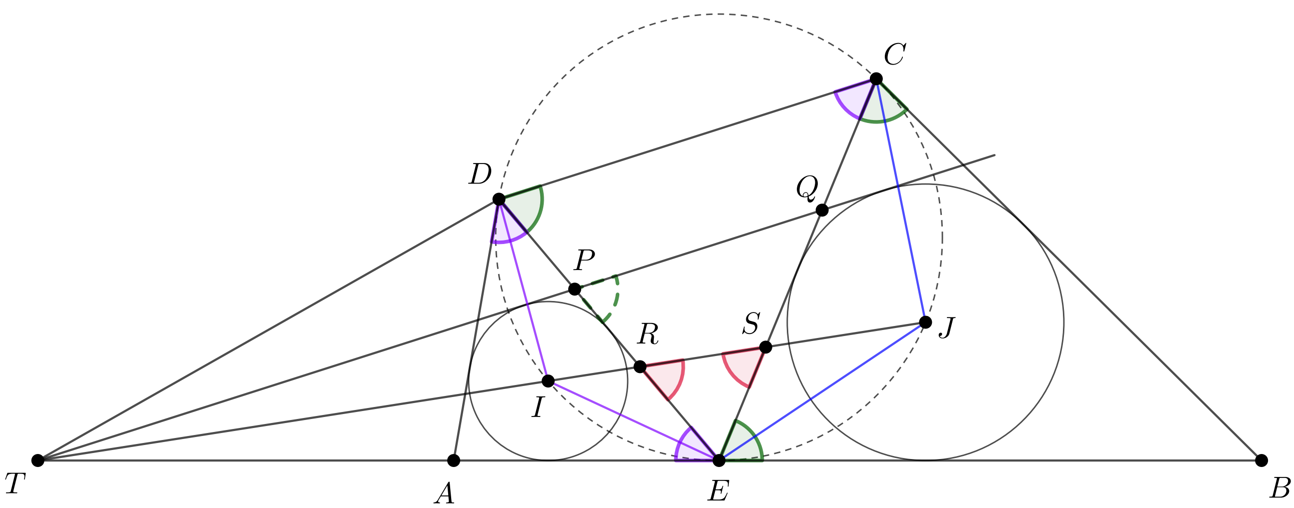
b) Suppose that the lines $PQ$, $AB$ and $IJ$ meet at a single point $T$. Then, by an angle chase, we get

$$|\sphericalangle EPQ| = |\sphericalangle ETP| + |\sphericalangle PET| = 2 \cdot |\sphericalangle ETR| + |\sphericalangle RET| = |\sphericalangle ETR| + (|\sphericalangle ETR| + |\sphericalangle RET|) =$$

$$|\sphericalangle ETR| + |\sphericalangle ERS| = |\sphericalangle ETS| + |\sphericalangle EST| = |\sphericalangle BEC| = |\sphericalangle EDC|.$$

Thus, in this case also, $PQ$ and $CD$ are parallel.

We have shown that in all cases $PQ$ and $CD$ are parallel, as we wanted to prove.



**Problem 6.**
Let $ABC$ be an acute scalene triangle with incenter $I$. Let $M$ be the midpoint of arc $BAC$ of the circumcircle of triangle $ABC$. Points $K$ and $L$ lie on segments $BM$ and $CM$, respectively, such that $AK$ is tangent to the circumcircle of triangle $AIC$ and $AL$ is tangent to the circumcircle of triangle $AIB$. Prove that the points $K$, $L$, $I$ are collinear.

*(Patrik Bak)*

**Solution**

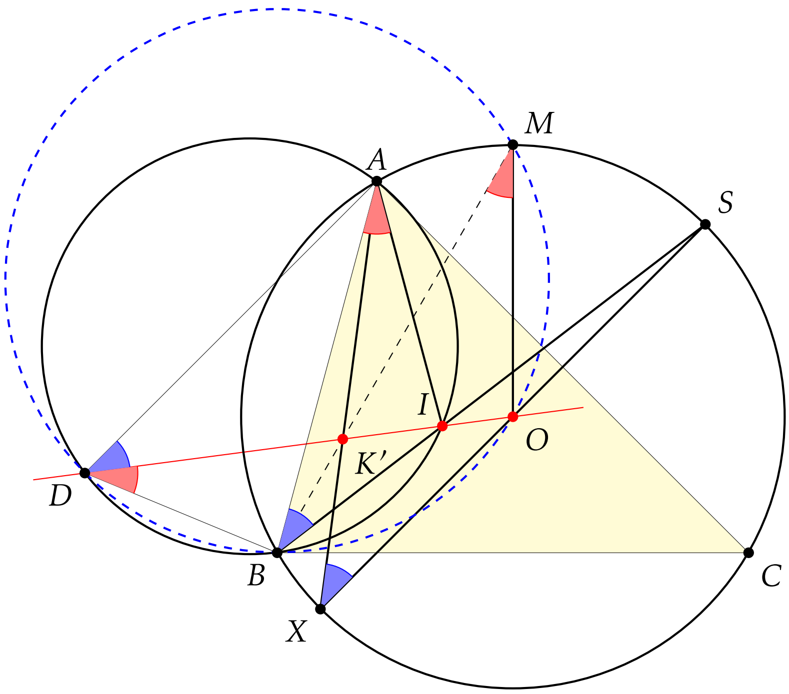
Let $O$ be the circumcenter of triangle $ABC$ and let $K'$ be the intersection of the line $IO$ with the tangent to the circumcircle of triangle $AIC$ at point $A$. We will prove that $K'$ lies on the line $BM$. This will imply that $K' = K$, and consequently, that points $K$, $I$, and $O$ are collinear. By analogous reasoning, it will follow that $L$, $I$, and $O$ are collinear, which will complete the proof.

Let $D$ be the second intersection of the line $OI$ and the circumcircle of triangle $AIB$. By a simple angle calculation, we get

$$|\sphericalangle BDO| = |\sphericalangle BDI| = |\sphericalangle BAI| = \frac{1}{2}|\sphericalangle BAC| = \frac{1}{2}|\sphericalangle BMC| = |\sphericalangle BMO|,$$

where we used that $O$ lies on the angle bisector of $\sphericalangle BMC$. As $|\sphericalangle BDO| = |\sphericalangle BMO|$, points $B$, $D$, $M$, $O$ lie on a circle.

Let $X$ be the second intersection of the tangent to the circumcircle of triangle $AIC$ at point $A$ and the circumcircle of triangle $ABC$. Let $S$ be the circumcenter of triangle $AIC$. It is well known that $S$ is the midpoint of arc $AC$ not containing $B$ of the circumcircle of triangle $ABC$. Since $XA$ is tangent to the circle $AIC$, we must have $AS \perp AX$, and therefore $X$, $O$, $S$ are collinear.

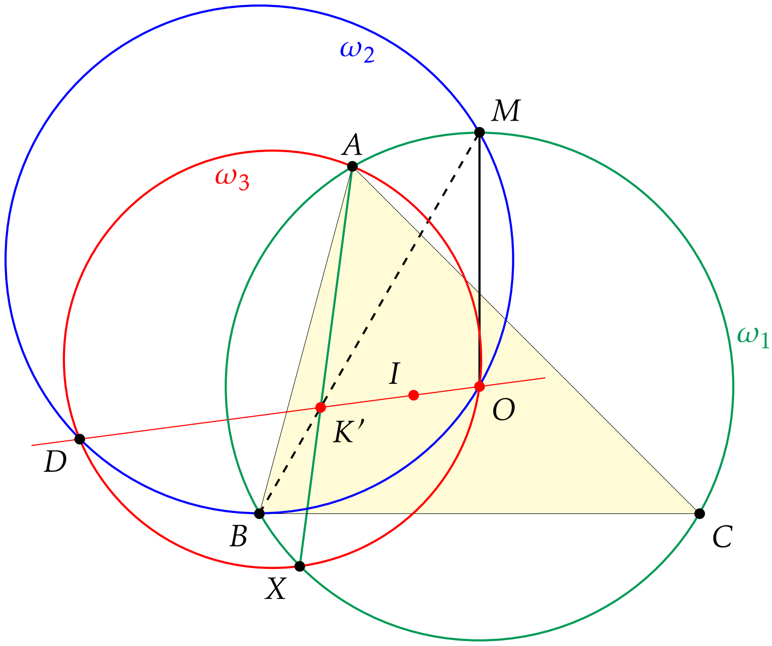



By an angle chase, we get

$$|\sphericalangle ADO| = |\sphericalangle ADI| = |\sphericalangle ABI| = |\sphericalangle ABS| = |\sphericalangle AXS| = |\sphericalangle AXO|,$$

hence we have that $A$, $D$, $X$, $O$ lie on a circle.

Denote by $\omega_1$ the circumcircle of triangle $ABC$, $\omega_2$ the circle through points $B$, $D$, $M$, $O$ and $\omega_3$ the circle through $A$, $D$, $X$, $O$. The line $AX$ is the radical axis of circles $\omega_1$ and $\omega_3$. The line $DO$ is the radical axis of circles $\omega_2$ and $\omega_3$. Therefore $K'$, which is the intersection of lines $AX$ and $DO$, is the radical center of circles $\omega_1$, $\omega_2$ and $\omega_3$. Hence, $K'$ also lies on the radical axis $BM$ of circles $\omega_1$ and $\omega_2$. Since $K'$ lies on $BM$, it follows that $K' = K$ and that $K$, $I$, $O$ are collinear, as discussed in the first paragraph.




**Solution II (Using Pascal's Theorem)**

Let $O$ be the circumcenter of triangle $ABC$. We will prove that points $K, O, I$ are collinear. Analogously, points $L, O, I$ are collinear, which completes the proof.

Let $N$ be the midpoint of the arc $BC$ not containing $A$ of the circumcircle of triangle $ABC$. Clearly, points $M$, $O$, $N$ are collinear, and points $A$, $I$, $N$ are collinear.

Let $S$ be the circumcenter of $AIC$. It is well known that it is the midpoint of the arc $AC$ not containing $B$ and lies on the line $BI$. Let $X$ be the second intersection of the line $AK$ with the circumcircle of $\triangle ABC$. Since $XA$ is tangent to the circle $AIC$, we must have $AS \perp AX$, and therefore $X$, $O$, $S$ are collinear.

Now we can apply Pascal's theorem to the self-intersecting hexagon $AXSBMN$ to obtain that the intersection points of pairs of lines $(AX, BM)$, $(XS, MN)$, and $(SB, NA)$ are collinear. These are points $K$, $O$, $I$, respectively, so we are done.